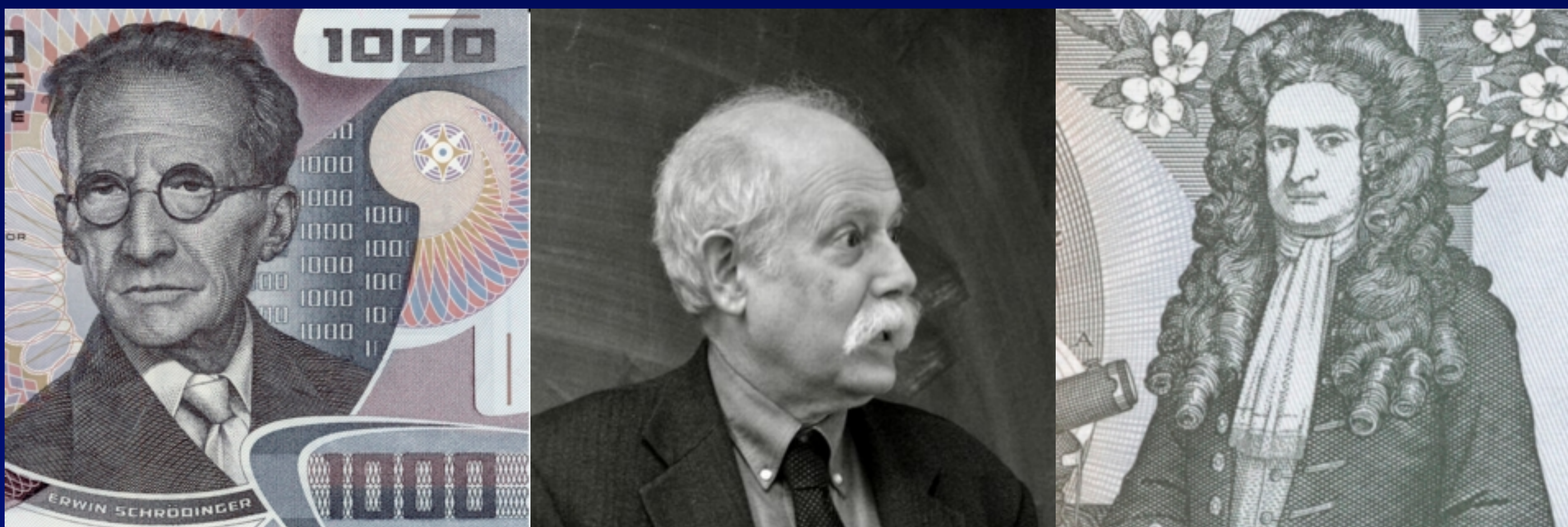
Jon the Quantum Mechanic

Chris Quigg
Fermi National Accelerator Laboratory



Jon Rosner Symposium · University of Chicago · 1 April 2011

First DPF Meeting · Boulder

Program for the Discussion Session On

Quark Models

Discussion Leader:
P. Freund

Date and time: 2:15-5:15 p.m., August 19, 1969
Place: West Ballroom

P. G. O. Freund - Hadron Dynamics and Quarks

J. L. Rosner - Quarks and Crossing Symmetry

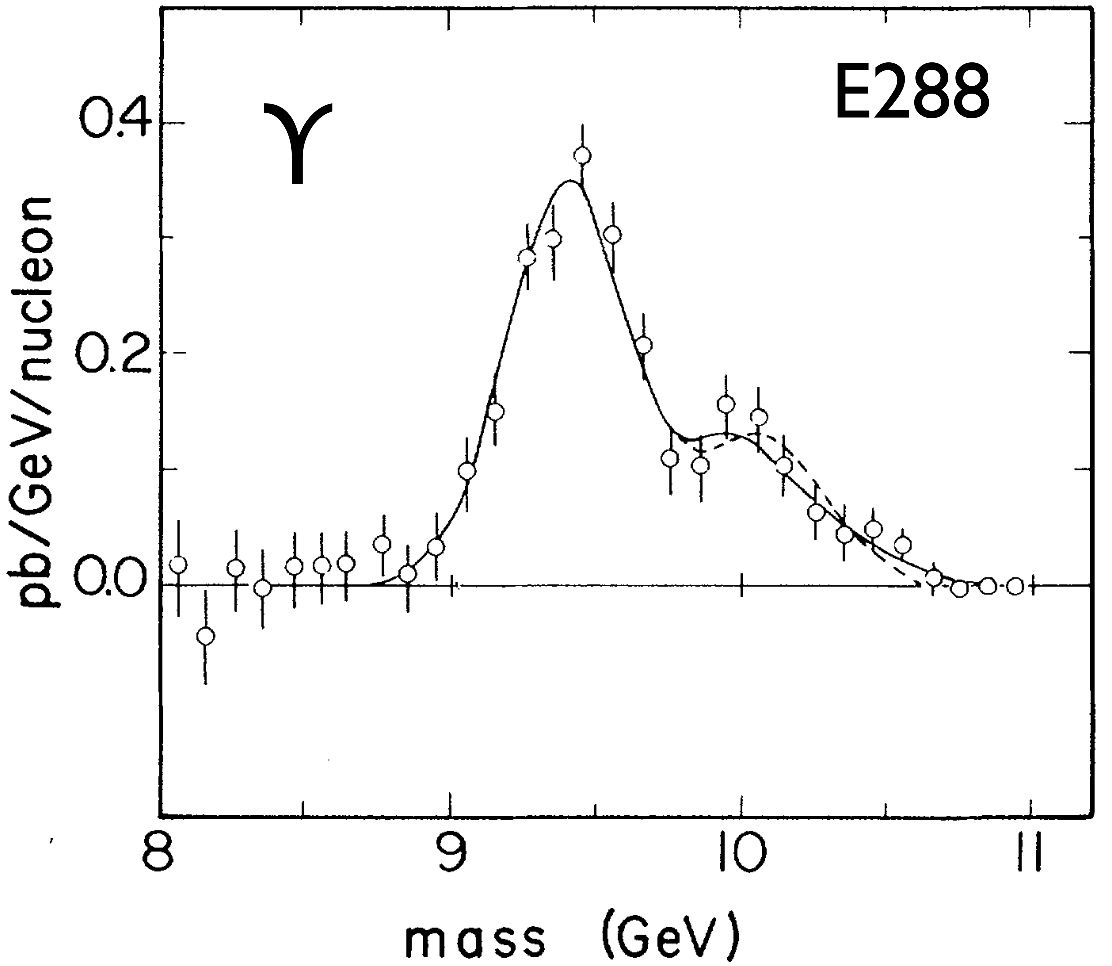
G. Zweig - A Model for the Hadrons

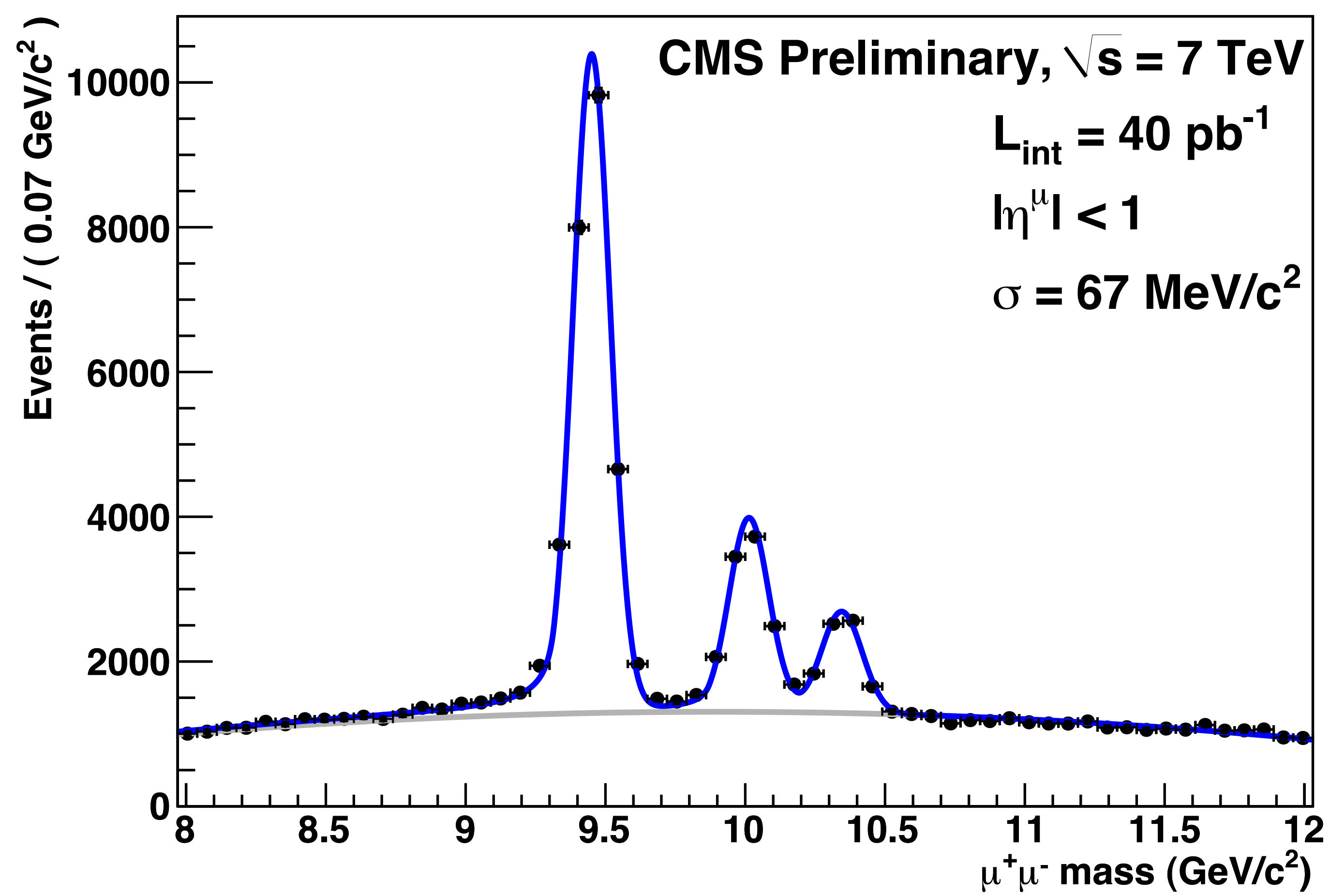
R. H. Capps - Baryonic Exchange Degeneracies

H. J. Lipkin - Algebraic Structure and Spin Couplings
In the Levin-Frankfurt Model
(No paper submitted)

S. Meshkov - Experimental Consequences of $SU(6)_W$

June 16, 1977: News of Υ





Quarkonium? One new quark or two?

Properties of new quark?

Is strong interaction flavor-blind?

The Empirical Approach

- Appelquist & Politzer: NRQM applies to $Q\bar{Q}$ systems.
- Cornell group: had shown the predictive power of the NR potential-model approach using a "culturally determined" potential,

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2} \quad .$$

- Eichten & Gottfried anticipated spectroscopy of $b\bar{b}$:

$$\begin{aligned} M(\Upsilon') - M(\Upsilon) &\approx 420 \text{ MeV} \\ &\approx \tfrac{2}{3}[M(\psi') - M(\psi)] \quad . \end{aligned}$$

$\Upsilon' - \Upsilon$ spacing same as $\psi' - \psi$

E288	$M(\Upsilon') - M(\Upsilon)$	$M(\Upsilon'') - M(\Upsilon)$
Two-level fit	650 ± 30 MeV	
Three-level fit	610 ± 40 MeV	1000 ± 120 MeV
$M(\psi') - M(\psi)$	≈ 590 MeV	

$$V(r) = C \log r \rightsquigarrow \Delta E \text{ independent of } \mu$$

Scaling the Schrödinger Equation: $V(r) = \lambda r^\nu$

$$\frac{\hbar^2}{2\mu} u'' + \left[E - \lambda r^\nu - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] u(r) = 0 \quad .$$

Substitute

$$r = \rho \left(\frac{\hbar^2}{2\mu|\lambda|} \right)^{1/(2+\nu)}$$

$$E = \varepsilon \left(\frac{\hbar^2}{2\mu|\lambda|} \right)^{-2/(2+\nu)} \left(\frac{\hbar^2}{2\mu} \right)$$

and identify $w(\rho) \equiv u(r)$: $\rightsquigarrow$ dimensionless form,

$$w''(\rho) + \left[\varepsilon - \mathrm{sgn}(\lambda)\rho^\nu - \frac{\ell(\ell+1)}{\rho^2} \right] w(\rho) = 0 \quad .$$

Length scale $[L] \propto (\mu|\lambda|)^{-1/(2+\nu)}$

Coulomb:	$(\mu\lvert\lambda\rvert)^{-1}$
Log: $V(r) = C \log r$	$(C\mu)^{-1/2}$
Linear:	$(\mu\lvert\lambda\rvert)^{-1/3}$
SHO:	$(\mu\lvert\lambda\rvert)^{-1/4}$
Square well:	$(\mu\lvert\lambda\rvert)^{0}$

Energies $[\Delta E] \propto (\mu)^{-\nu/(2+\nu)}(|\lambda|)^{2/(2+\nu)}$

Coulomb:	$\mu\lvert\lambda\rvert^{2}$
Log: $V(r) = C \log r$	$C\mu^{0}$
Linear:	$\mu^{-1/3}\lvert\lambda\rvert^{2/3}$
SHO:	$\mu^{-1/2}\lvert\lambda\rvert^{1/2}$
Square well:	μ^{-1}

Measuring the b-quark's charge

For $\nu \leq 1$, scaling laws imply

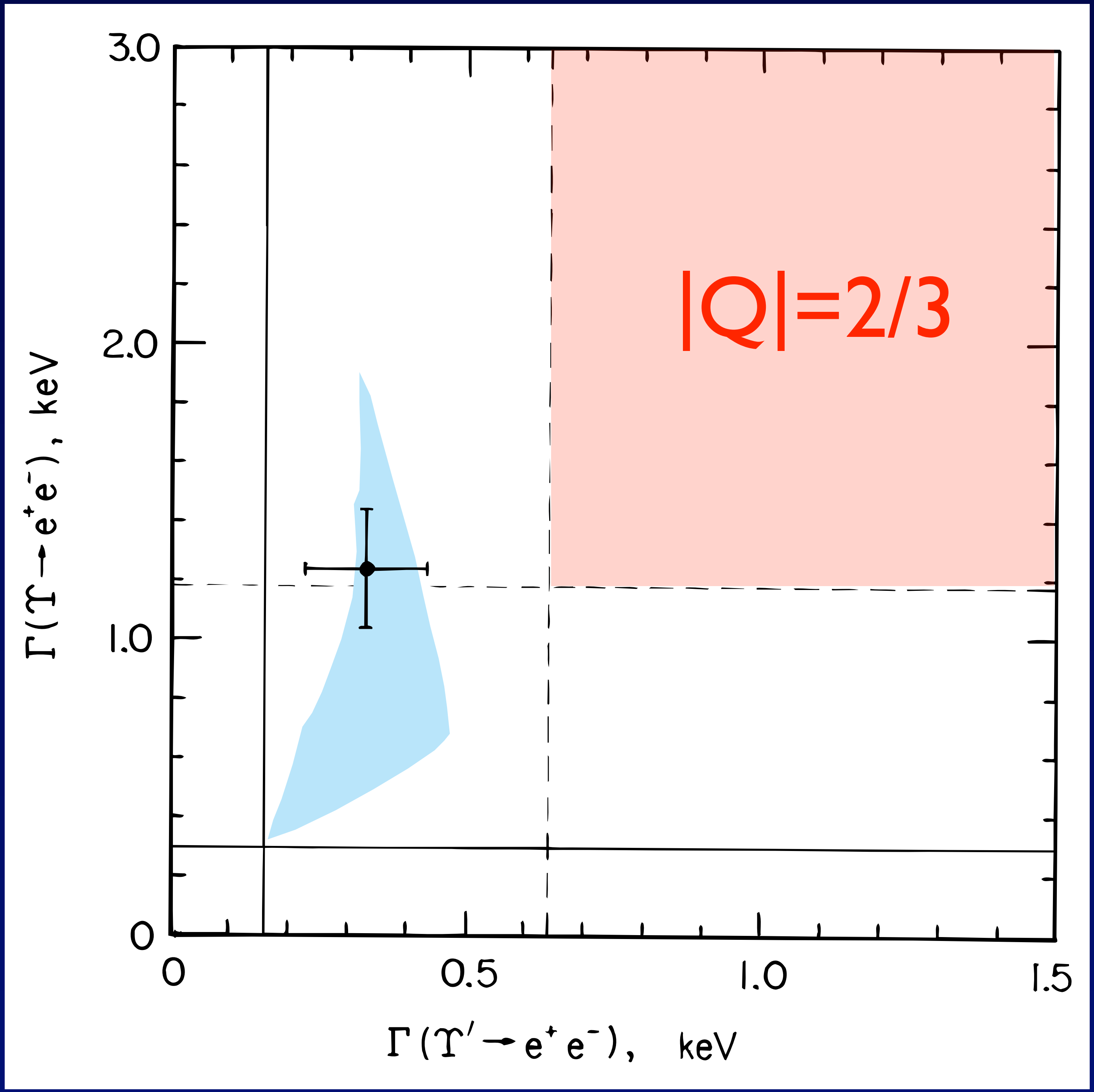
$$|\Psi_b(0)|^2 \geq \frac{m_b}{m_c}|\Psi_c(0)|^2$$

$$\leadsto \Gamma(\Upsilon_n \to \ell^+\ell^-) \geq \frac{e_b^2}{e_c^2} \cdot \frac{m_b}{m_c} \cdot \frac{M(\psi_n)^2}{M(\Upsilon_n)^2}\Gamma(\psi_n \to \ell^+\ell^-)$$

DORIS results presented at 1978 ICHEP (Tokyo)

$$\begin{array}{ll} \Gamma(\Upsilon \to \ell^+\ell^-) & 1.26 \pm 0.21 \text{ keV} \\ \Gamma(\Upsilon' \to \ell^+\ell^-) & 0.36 \pm 0.09 \text{ keV} \end{array}$$

established $e_b = -\frac{1}{3}$

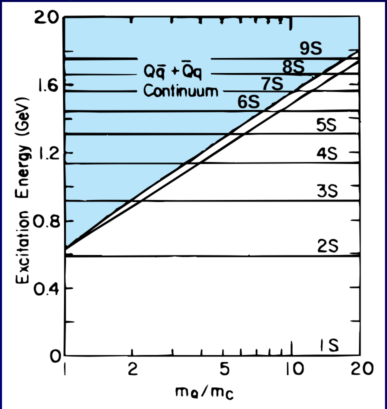


|Q|=2/3
3.0
2.0
1.0
0
0
0.5
1.0
1.5
Γ(Υ→e⁺e⁻), keV
Γ(Υ′→e⁺e⁻), keV

Counting Narrow Levels of Quarkonium

Eichten–Gottfried: $Q\bar{Q}$ ($m_Q \gg m_c$): ≥ 3 narrow $^3S_1$ levels, but $\Upsilon' - \Upsilon$ spacing is much larger than predicted.

How general? Depends on?



Logarithmic potential: 3 or 4 narrow $\Upsilon(^3S_1)$ levels.

Counting Narrow Levels . . .

Remarkable general result

Number of narrow $^3S_1$ levels is

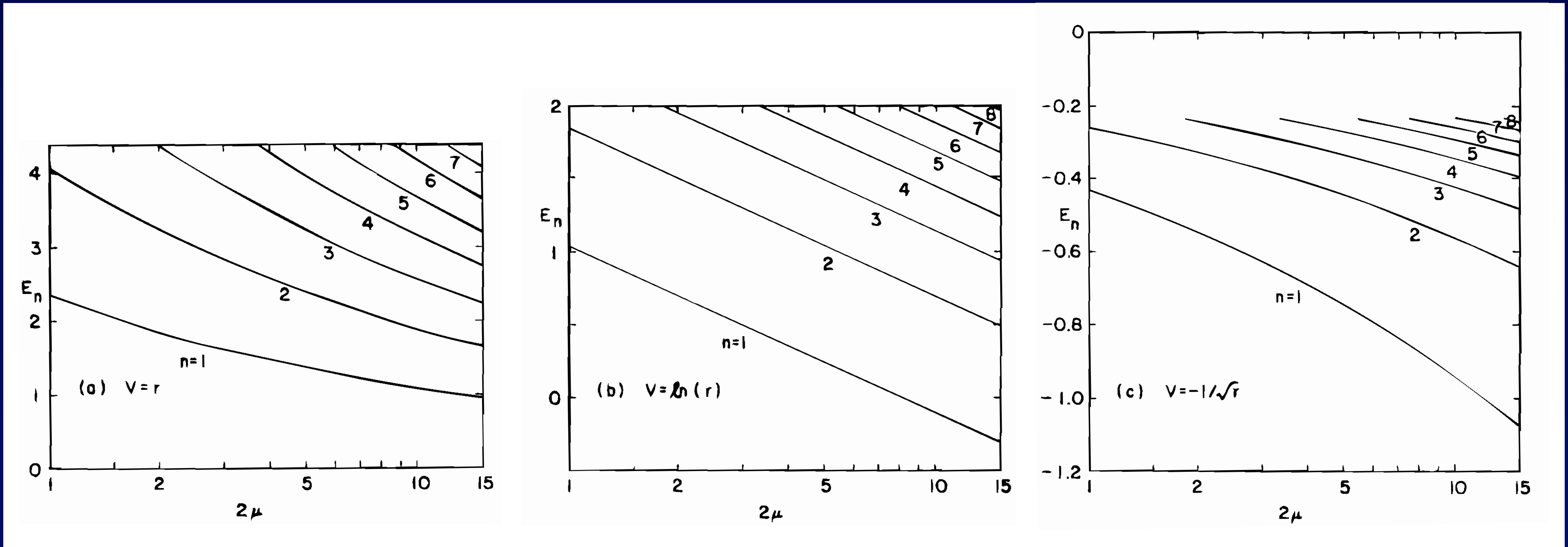
$$n \approx 2 \cdot \left(\frac{m_Q}{m_c}\right)^{1/2}$$

Key ingredients:

- $\delta(m_Q) \equiv 2m(\text{lowest } \; Q\bar{q} \; \text{ state}) - 2m_Q$ approaches a finite δ_∞ independent of m_Q.
- Semiclassical (WKB) approximation:

$$\int_0^{r_0} dr \, [m_Q(\delta(m_Q) - V(r))]^{1/2} = (n - \tfrac{1}{4})\pi$$

How n $\propto \sqrt{m_Q}$ is realized



Semiclassical Methods and Results

Evaluating the nonrelativistic connection

$$|\Psi_n(0)|^2 = \frac{\mu}{2\pi}\left\langle \frac{dV}{dr} \right\rangle_n$$

in semiclassical approximation, connect the square of the *s*-wave wave function at the origin to the level density:

$$|\Psi_n(0)|^2 = \frac{(2\mu)^{3/2}}{4\pi^2} E_n^{1/2} \frac{dE_n}{dn}$$

(for a nonsingular potential).

Elementary application

For $V(r) = \lambda r$, $|\Psi_n(0)|^2$: independent of n, $\rightsquigarrow E_n \sim n^{2/3}$.

Semiclassical Inverse Problem

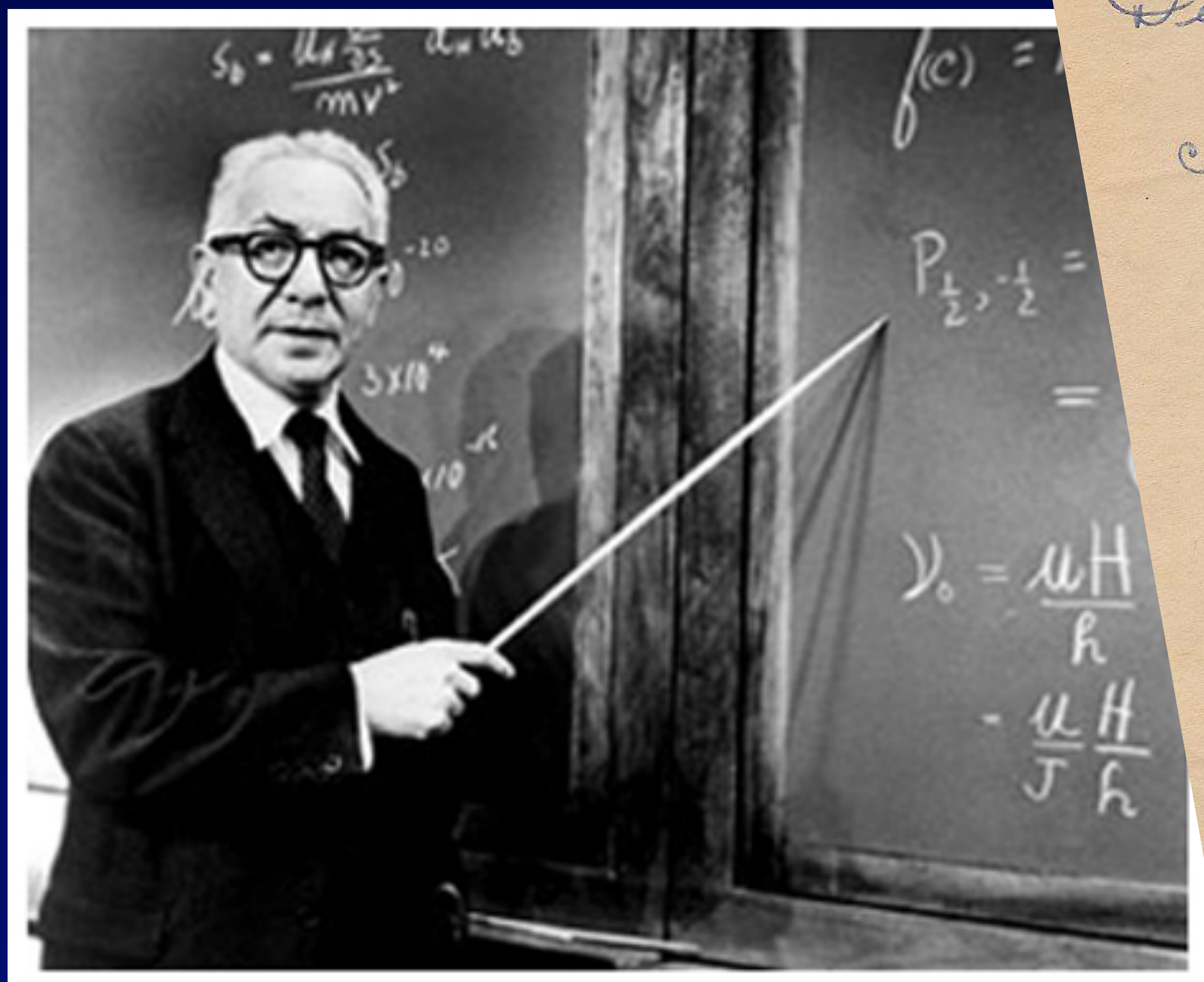
For a monotonically increasing potential, the semiclassical quantization condition

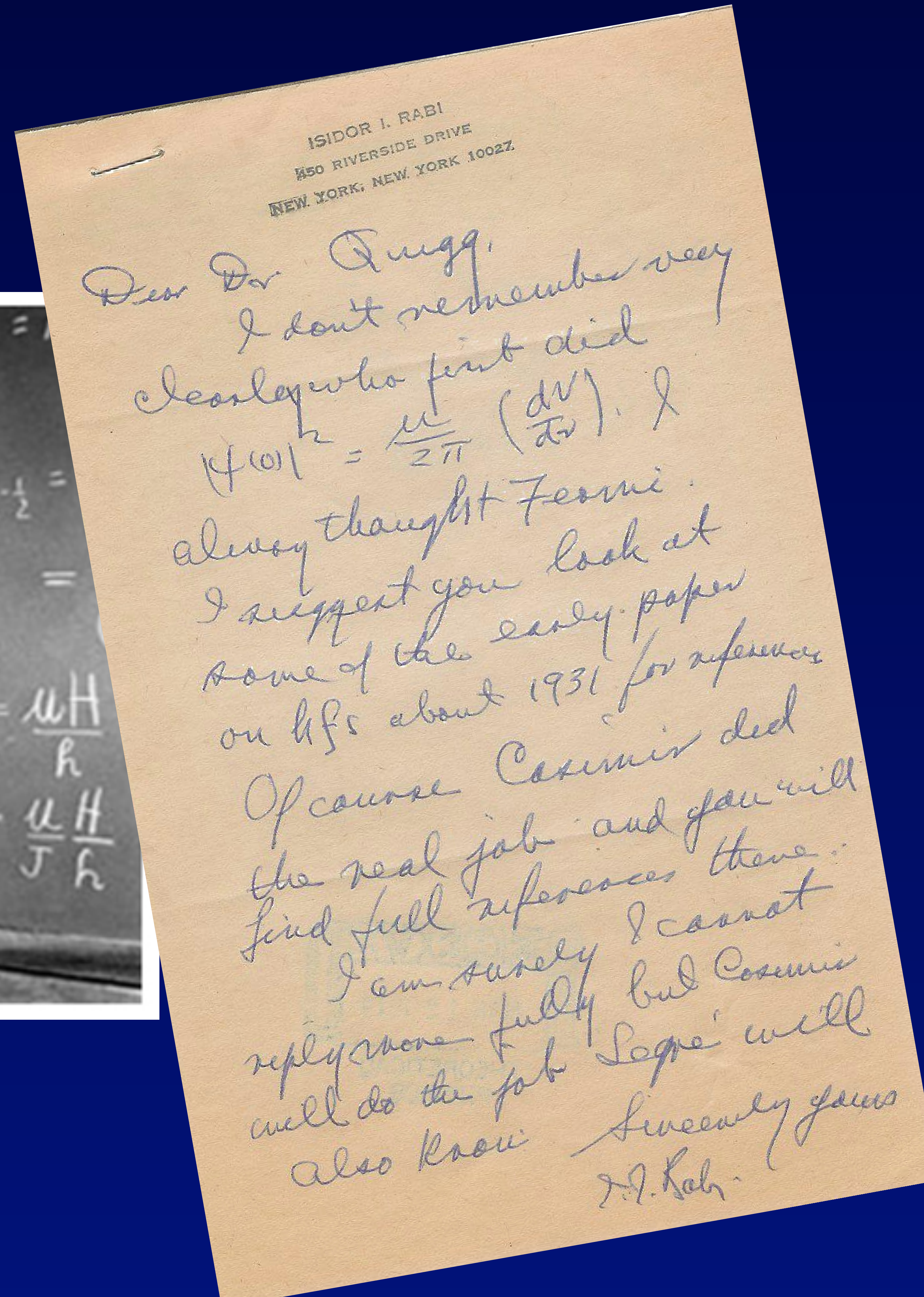
$$\int_0^{r_0} dr\{2\mu[E - V(r)]\}^{1/2} = (n - \tfrac{1}{4})\pi$$

connects the shape of the potential to the level density:

$$r(V) = \frac{2}{2\mu^{1/2}} \int_0^V dE(V - E)^{1/2} \left[\frac{dE_n}{dn}\right]^{-1} \quad .$$

New results, or results forgotten for 40 years?

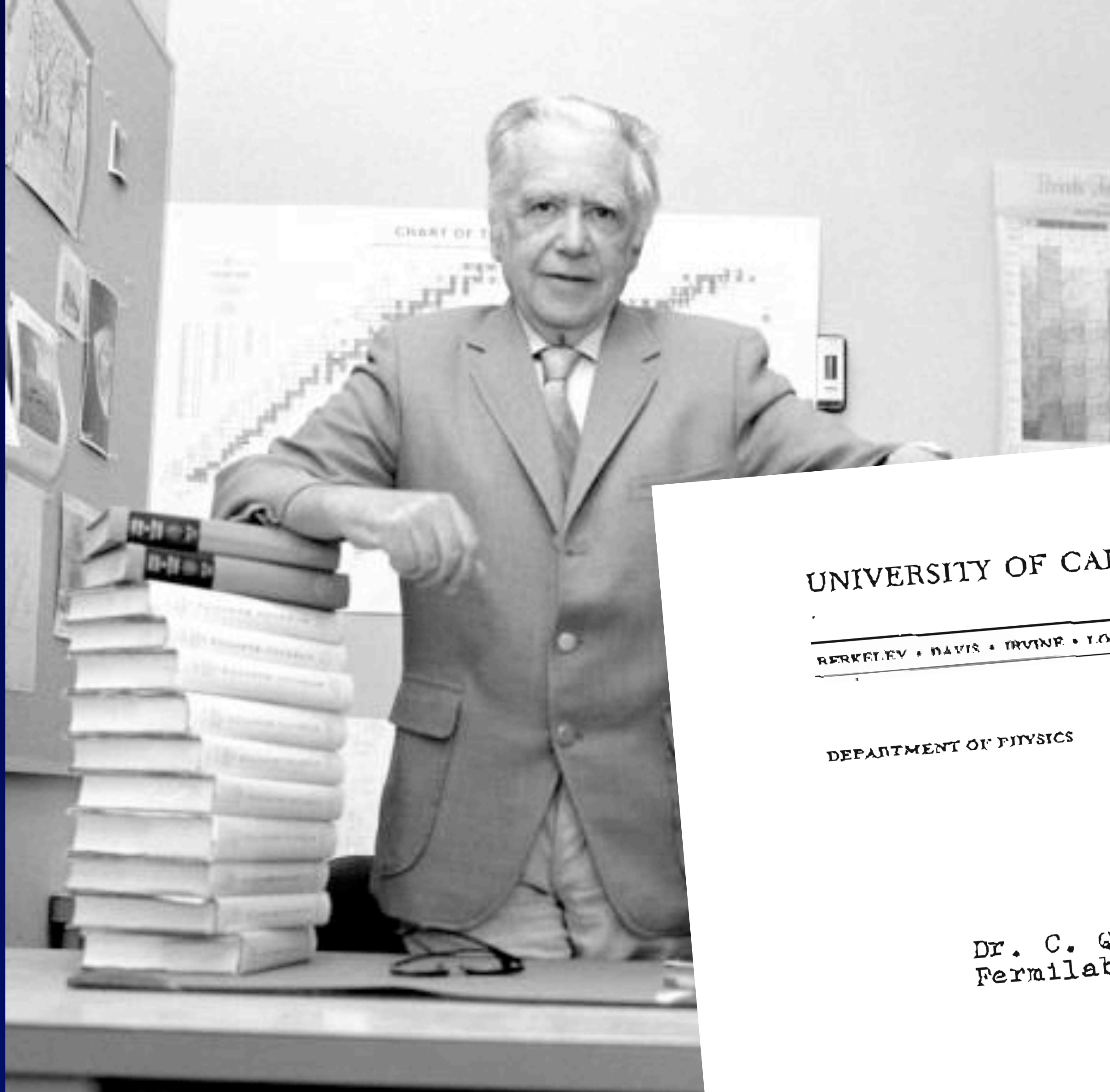




ISIDOR I. RABI
450 RIVERSIDE DRIVE
NEW YORK, NEW YORK 10027

Dear Dr. Quigg,
I don't remember very clearly who first did
$|\psi(0)|^2 = \frac{\mu}{2\pi}\left(\frac{dV}{dn}\right)$. I always thought Fermi.
I suggest you look at some of the early paper on hfs about 1931 for reference.
Of course Casimir did the real job and you will find full references there.
I am surely I cannot reply more fully but Casimir will do the job. Segrè will also know

Sincerely yours
I. I. Rabi



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DEPARTMENT OF PHYSICS

BERKELEY, CALIFORNIA 94720

March 30 1979

Dr. C. Quigg
Fermilab
Batavia IL

Dear Dr. Quigg:

In reply to your letter of March 19th. I do not remember having seen the formula for $|\psi(0)|^2$ that you quote. Fermi has had repeated occasions for calculating $\psi(0)$ and you may find it in papers # 75b, 95 of the Collected papers. It was a quantity very familiar to him and on which he returned often. However I do not recall the specific formula you quote. In what context did you obtain it ?

Sincerely yours

Emilio Segrè

Emilio Segre

ISSN 0 370 – 1573

PHYSICS REPORTS

A Review Section of Physics Letters

QUANTUM MECHANICS WITH APPLICATIONS TO QUARKONIUM

C. QUIGG and J.L. ROSNER

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N·H P·C

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Dualities

Connect bound-state spectra of $V(r) = \lambda r^{\nu} \;\; (\nu > 0)$ and $\bar{V}(r) = \bar{\lambda} r^{\bar{\nu}} \;\; -2 < (\bar{\nu} < 0)$

Paired Schrödinger equations

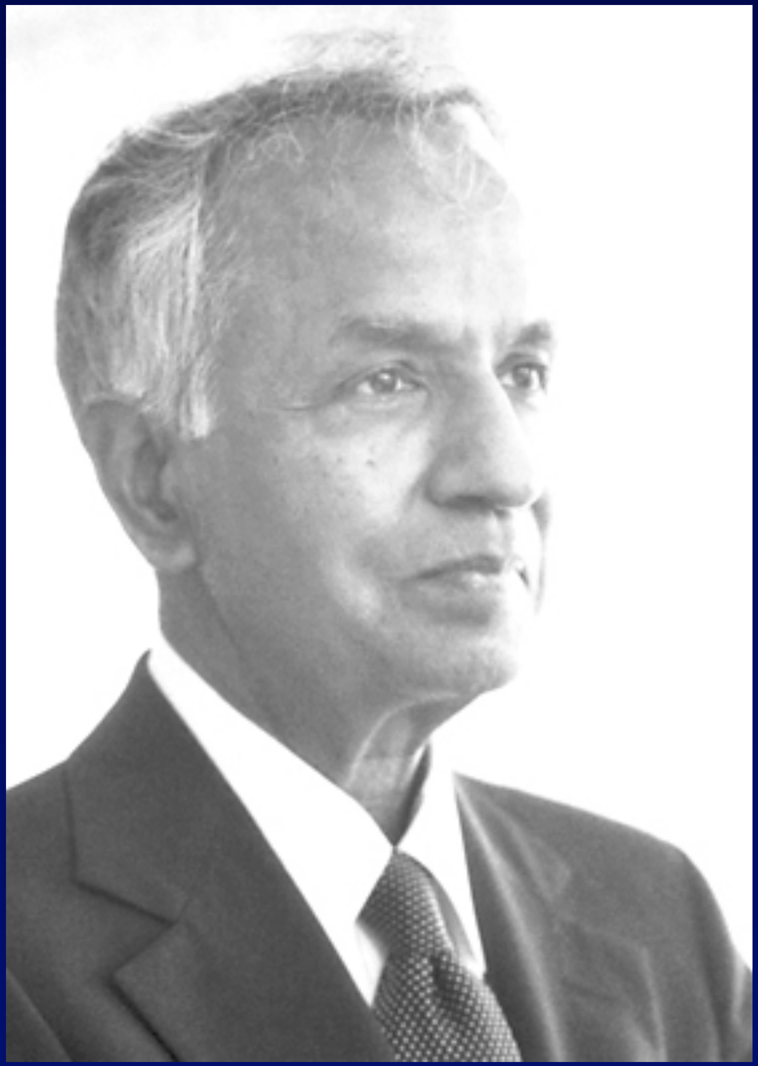
$$\begin{aligned} \frac{\hbar^2}{2\mu} u''(r) + \left[E - \lambda r^{\nu} - \frac{\ell(\ell+1)\hbar^2}{2\mu r^2} \right] u(r) &= 0 \\ \frac{\hbar^2}{2\mu} v''(z) + \left[\bar{E} - \bar{\lambda} z^{\bar{\nu}} - \frac{\bar{\ell}(\bar{\ell}+1)\hbar^2}{2\mu z^2} \right] v(z) &= 0 \end{aligned}$$

$$(\nu+2)(\bar{\nu}+2) = 4, \; \bar{E} = \lambda(\bar{\nu}/\nu)^2, \; \bar{\lambda} = -E(\bar{\nu}/\nu)^2,$$

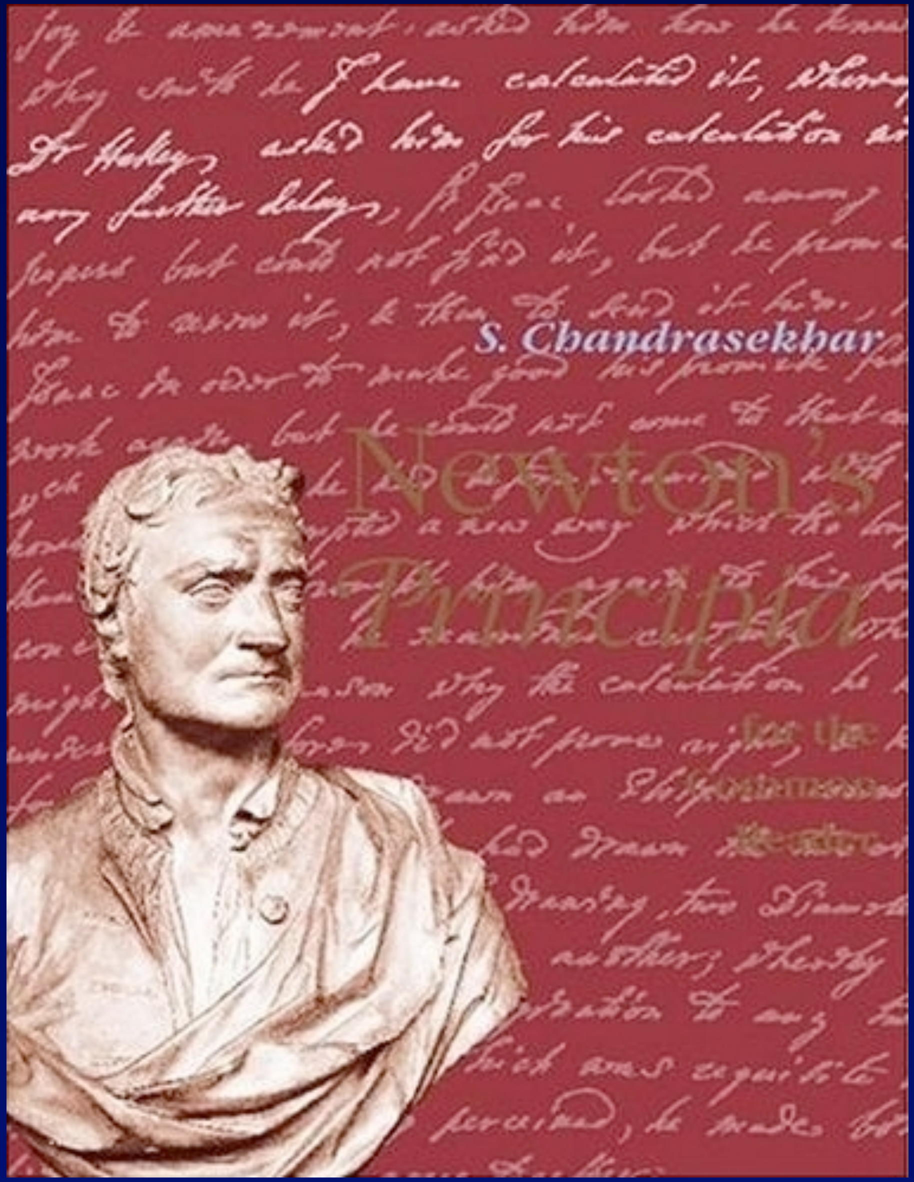
$$z = r^{1+\nu/2}, \; (\bar{\ell}+1/2)^2\nu^2 = (\ell+1/2)^2\bar{\nu}^2$$

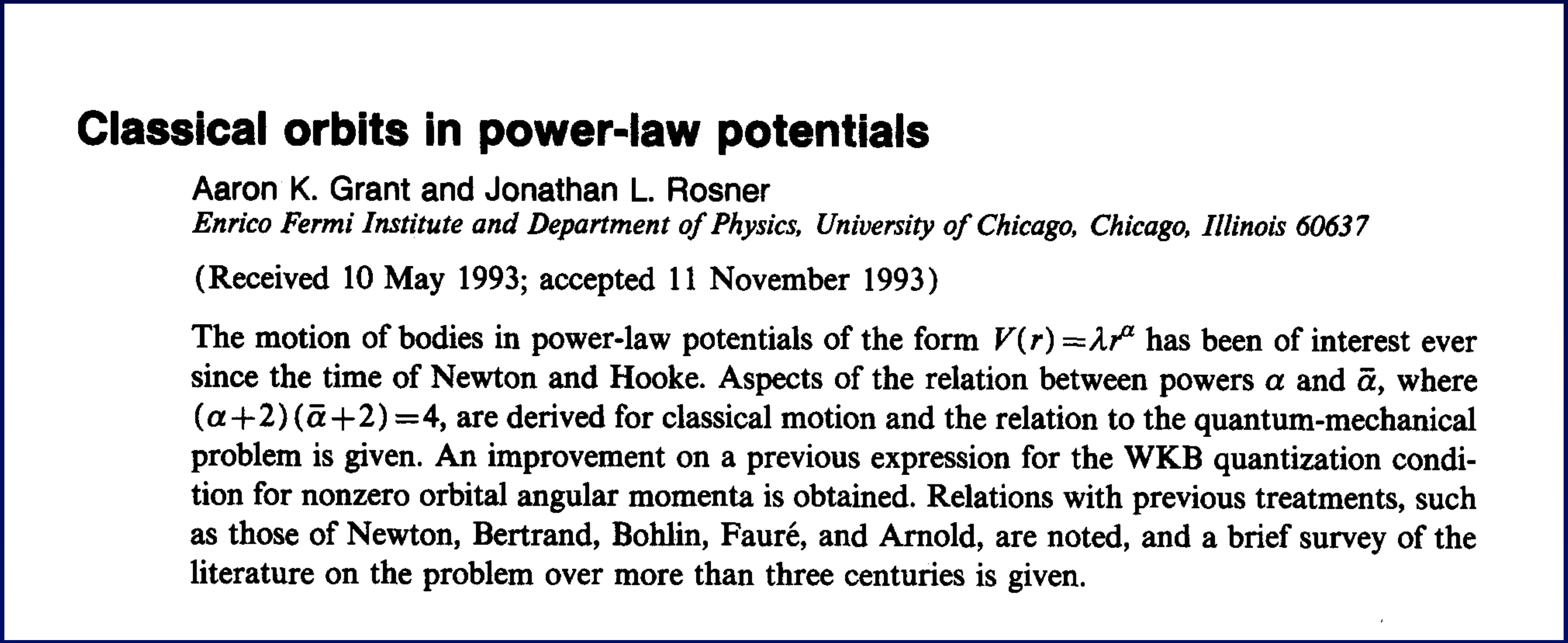
familiar case Coulomb $\Longleftrightarrow$ harmonic oscillator

Our Priority Dispute with Isaac Newton



S. Chandresekhar





Classical orbits in power-law potentials

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(Received 10 May 1993; accepted 11 November 1993)

The motion of bodies in power-law potentials of the form $V(r) = \lambda r^{\alpha}$ has been of interest ever since the time of Newton and Hooke. Aspects of the relation between powers α and $\bar{\alpha}$, where $(\alpha+2)(\bar{\alpha}+2)=4$, are derived for classical motion and the relation to the quantum-mechanical problem is given. An improvement on a previous expression for the WKB quantization condition for nonzero orbital angular momenta is obtained. Relations with previous treatments, such as those of Newton, Bertrand, Bohlin, Fauré, and Arnold, are noted, and a brief survey of the literature on the problem over more than three centuries is given.

Am. J. Phys. 62, 310 (1994)

Designer Potentials

Hank Thacker

Construct a symmetric, one-dimensional potential that supports N bound states at specified E_n out of reflectionless potentials

$$V(x) = -2\kappa^2 \mathrm{sech}^2[\kappa(x - x_0)]$$

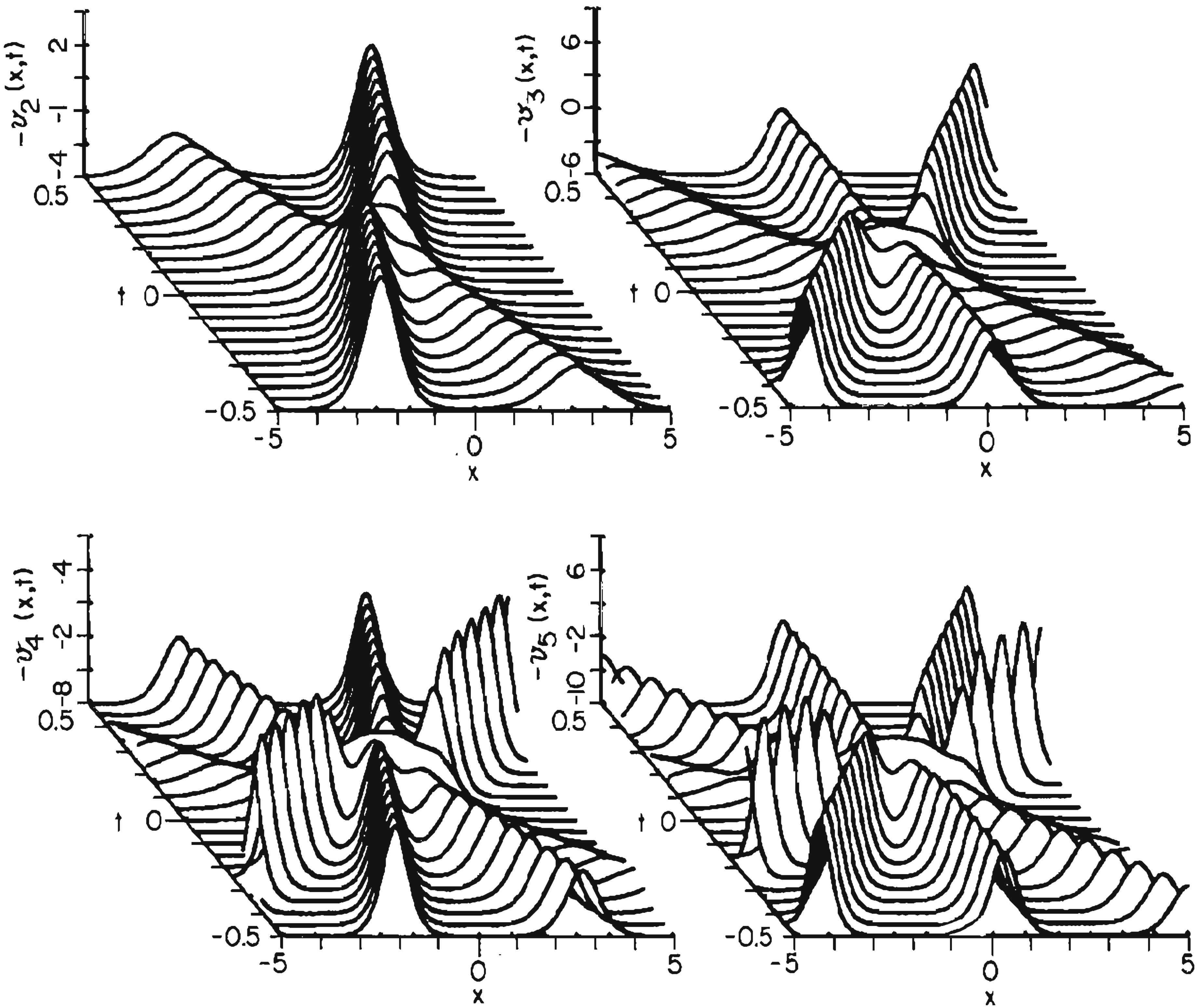
(single bound state at $E = -\kappa^2$)

N-level reflectionless potential:
N-solitary-wave solution to

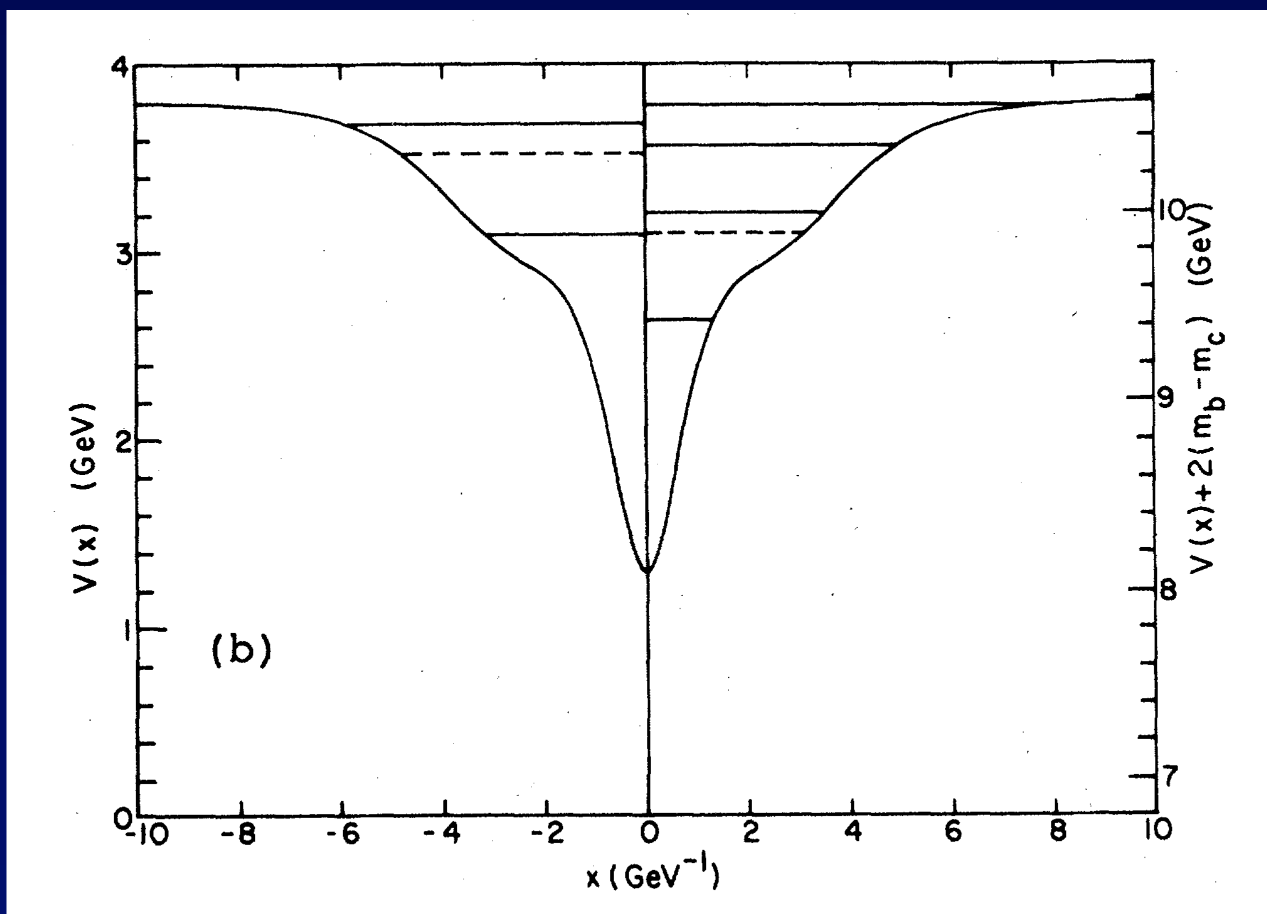
$$v_t - 6vv_x + v_{xxx} = 0$$

Korteweg-de Vries equation

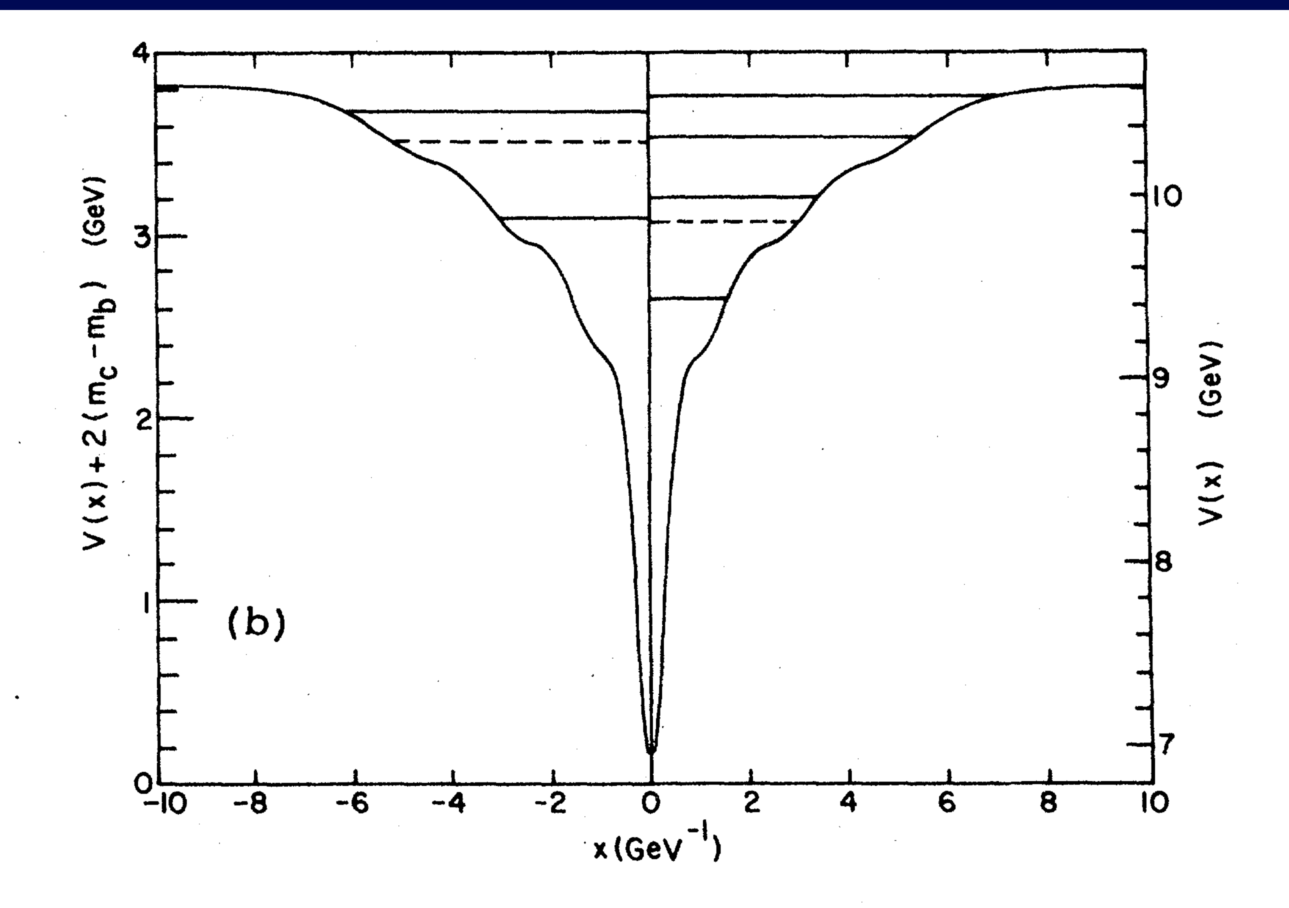
Reflectionless potentials as Korteweg-de Vries solitons: harmonic oscillator example



Flavor independence of strong interaction among heavy quarks



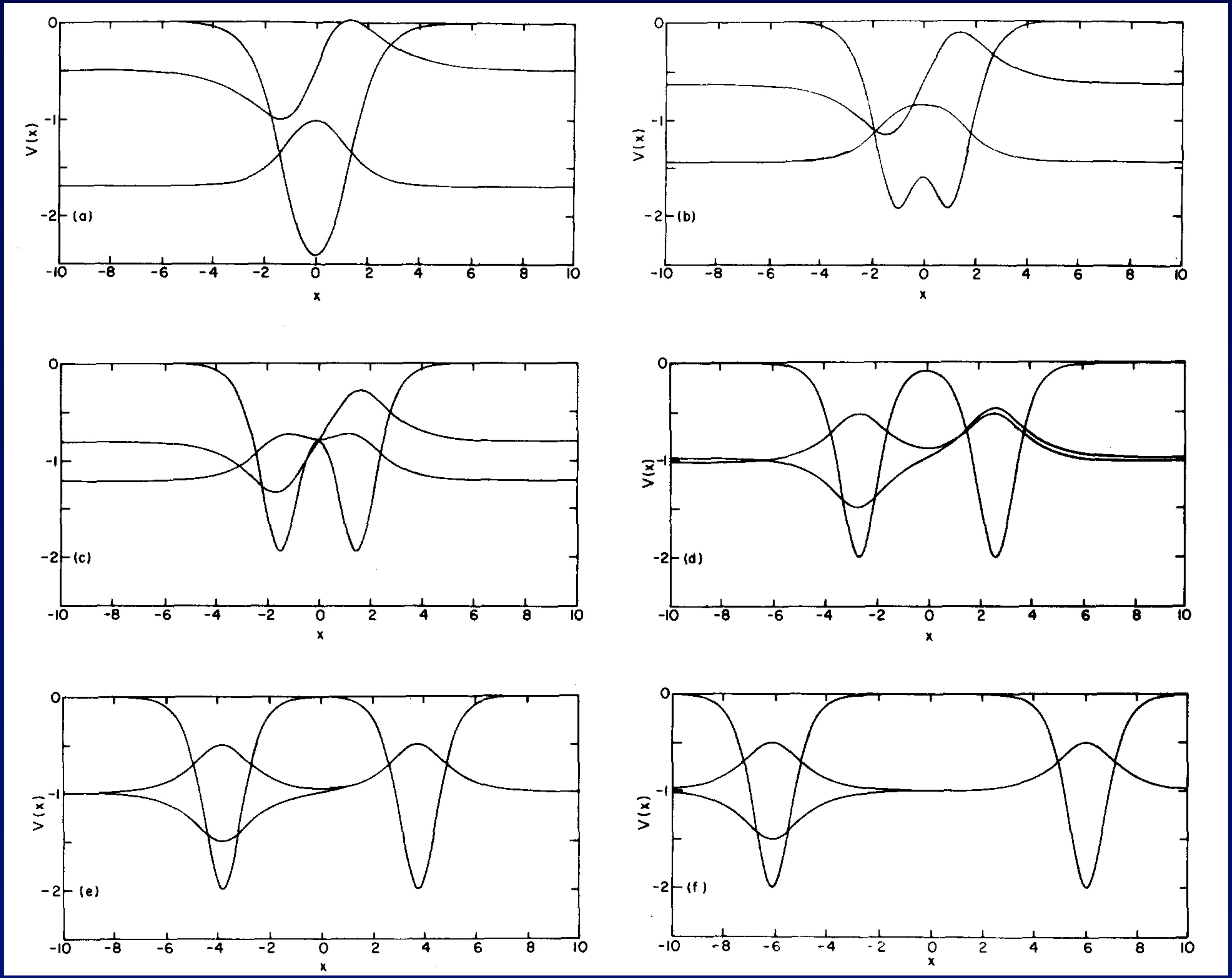
Reconstructed from ψ



Reconstructed from ϒ

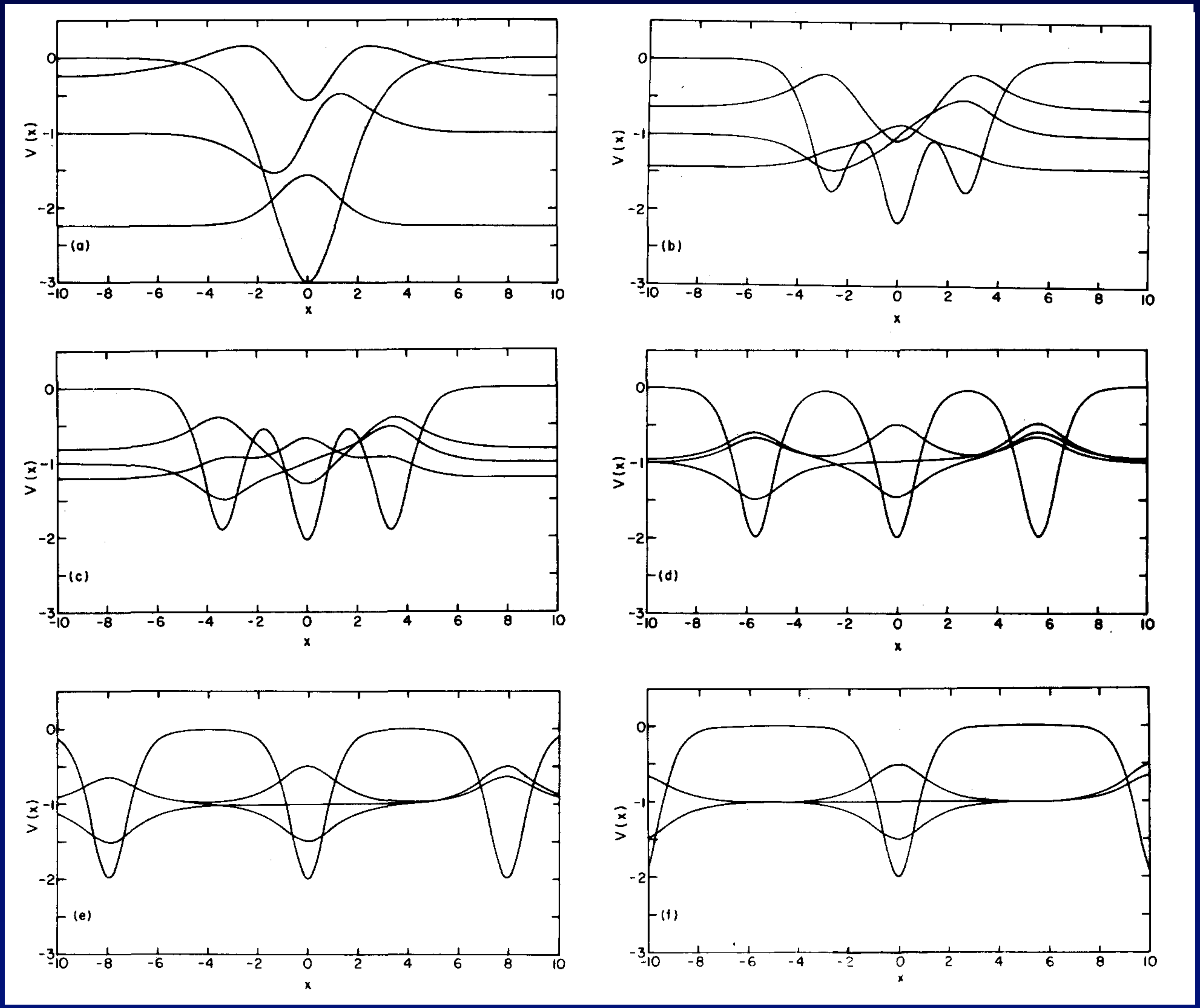
No Degenerate Levels in One-Dimensional QM

(simple Wronskian proof, if no pathologies)

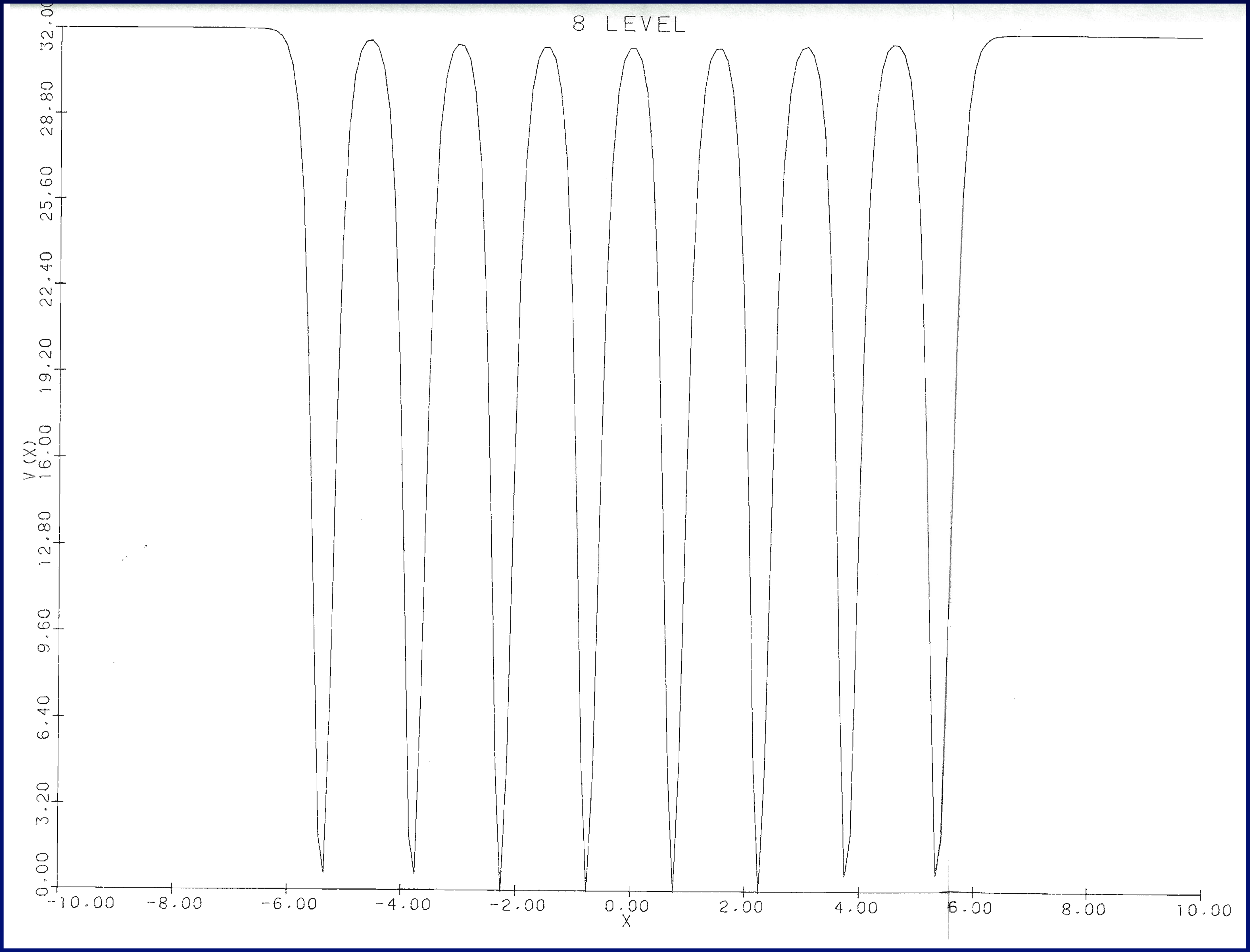


As levels approach, two buckets retreat to ±∞

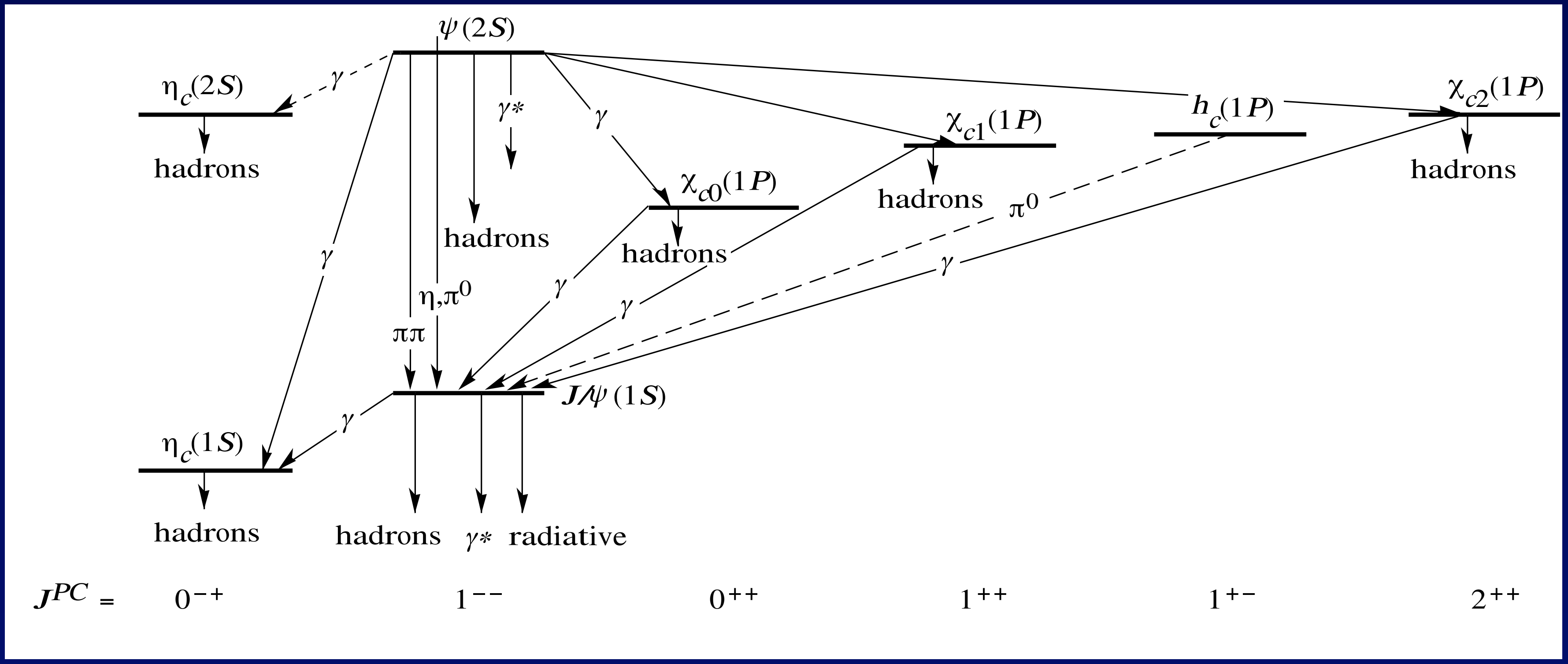
Band Structure and Periodic Potentials



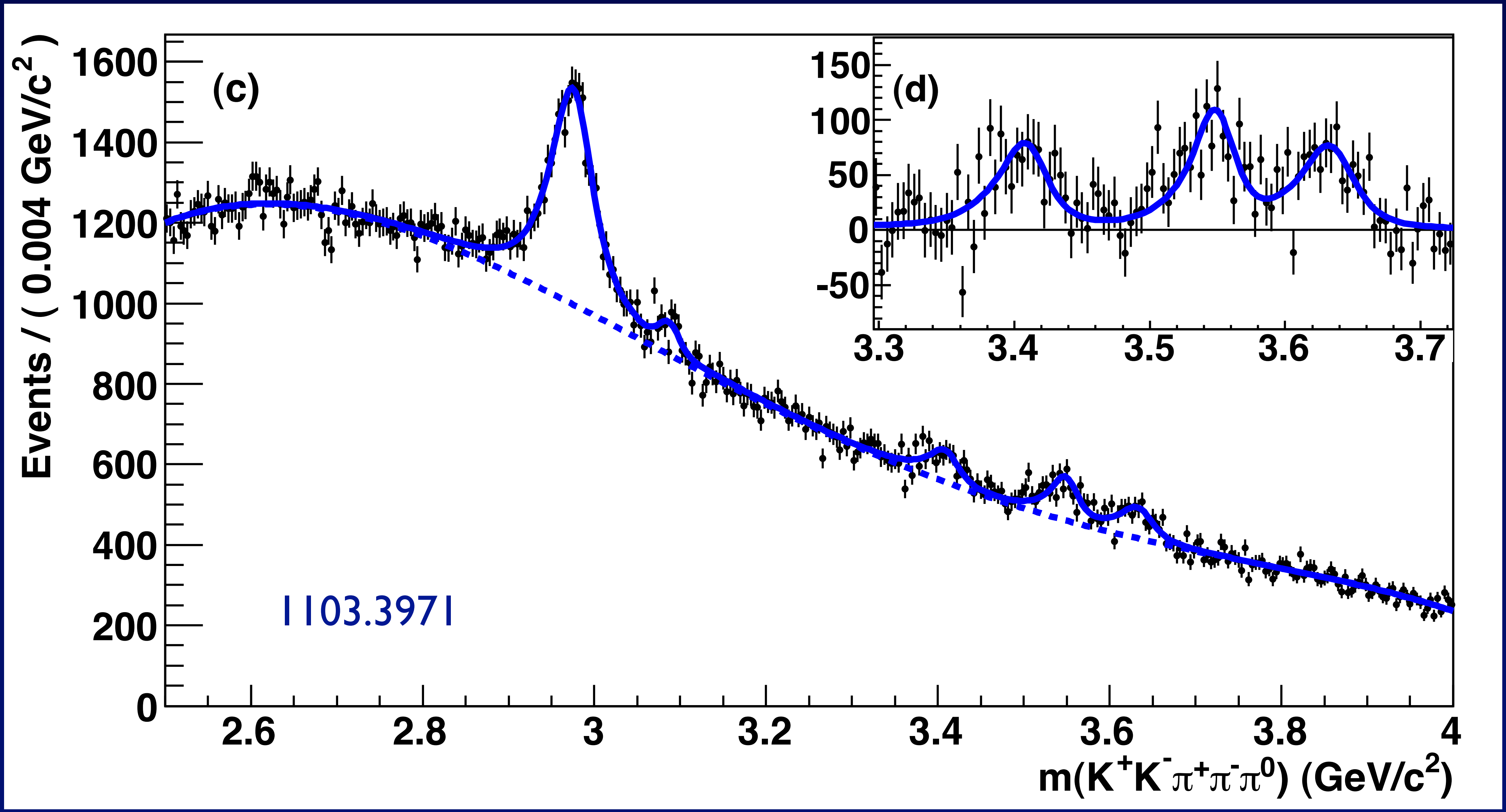
Band Structure and Periodic Potentials



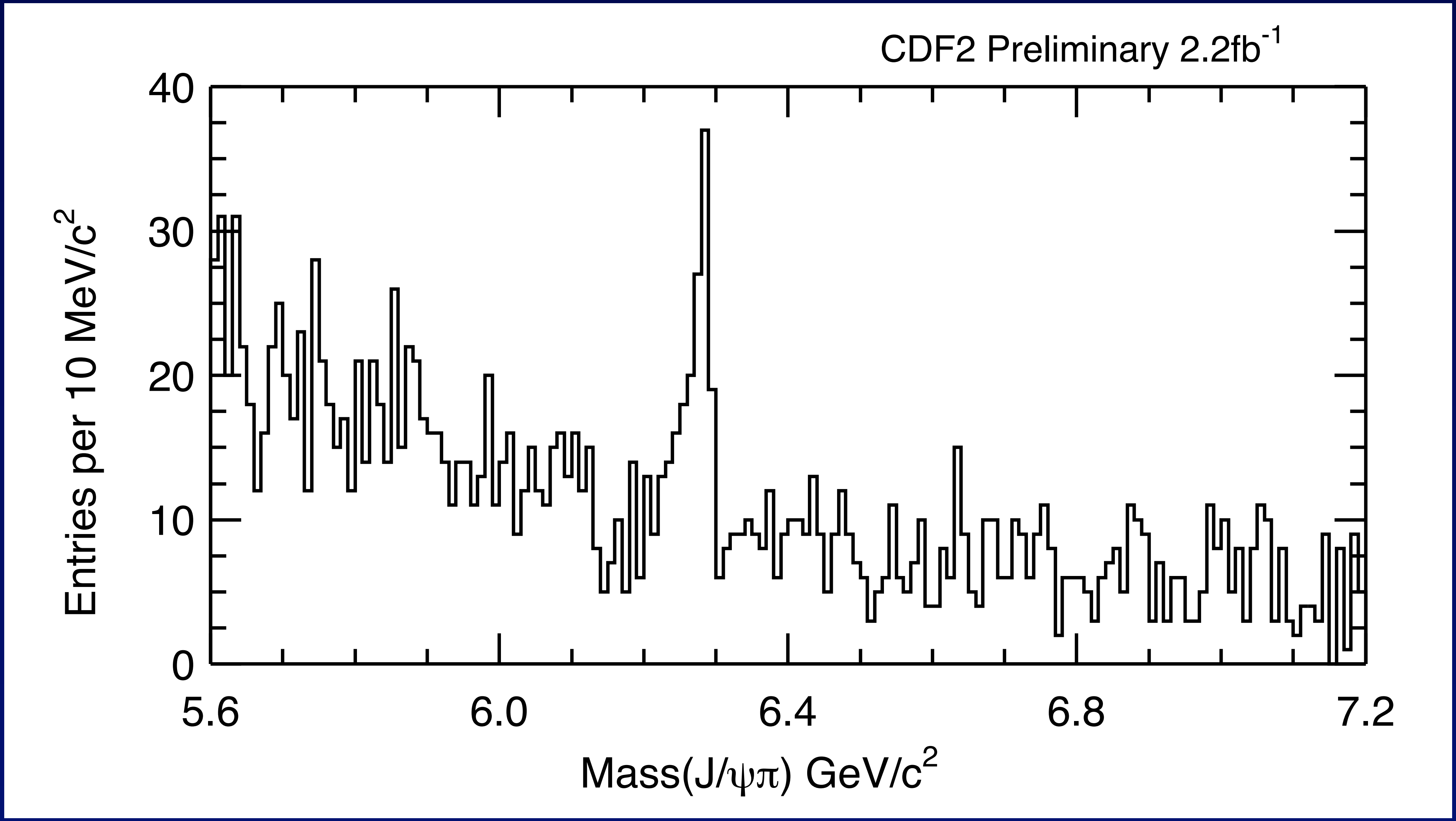
Charmonium (PDG)



BaBar $\eta_c(1S)$, $\chi_{c0}(1P)$, $\chi_{c2}(1P)$, $\eta_c(2S)$ in $\gamma\gamma$

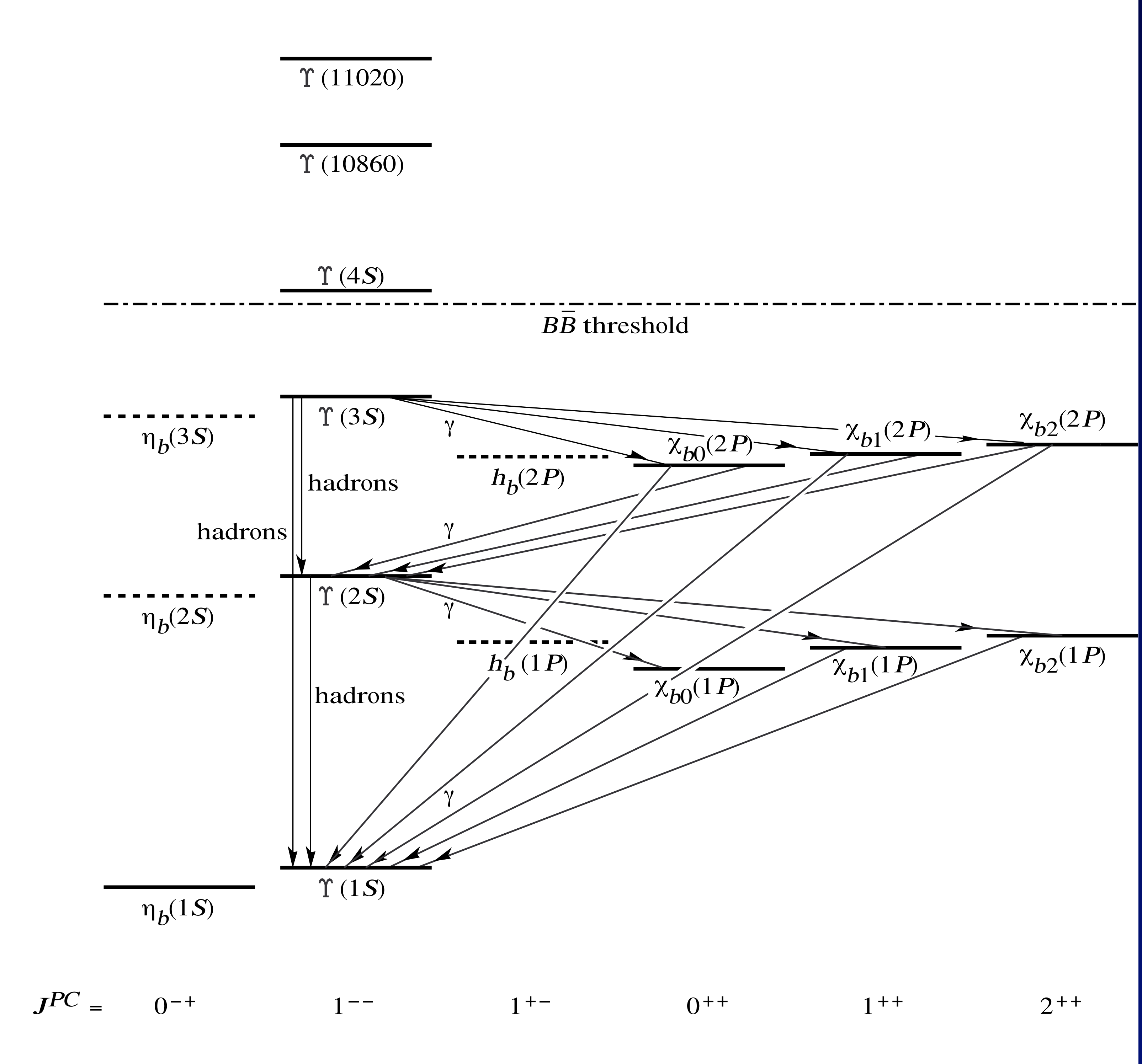


B_c (6277)

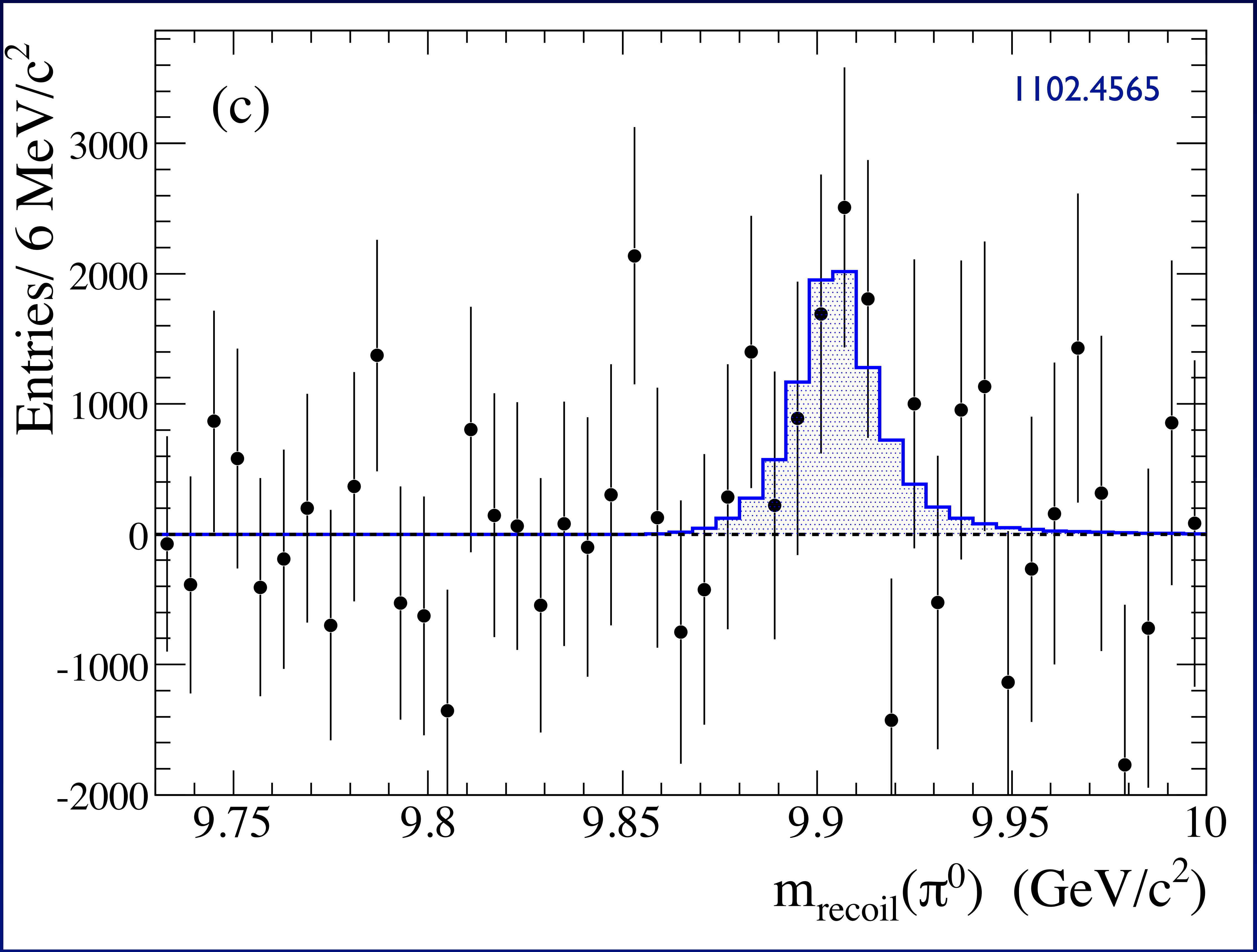


CDF2 Preliminary 2.2fb^{-1}
40
30
20
10
0
Entries per 10 MeV/c^2
5.6
6.0
6.4
6.8
7.2
Mass(J/ψπ) GeV/c^2

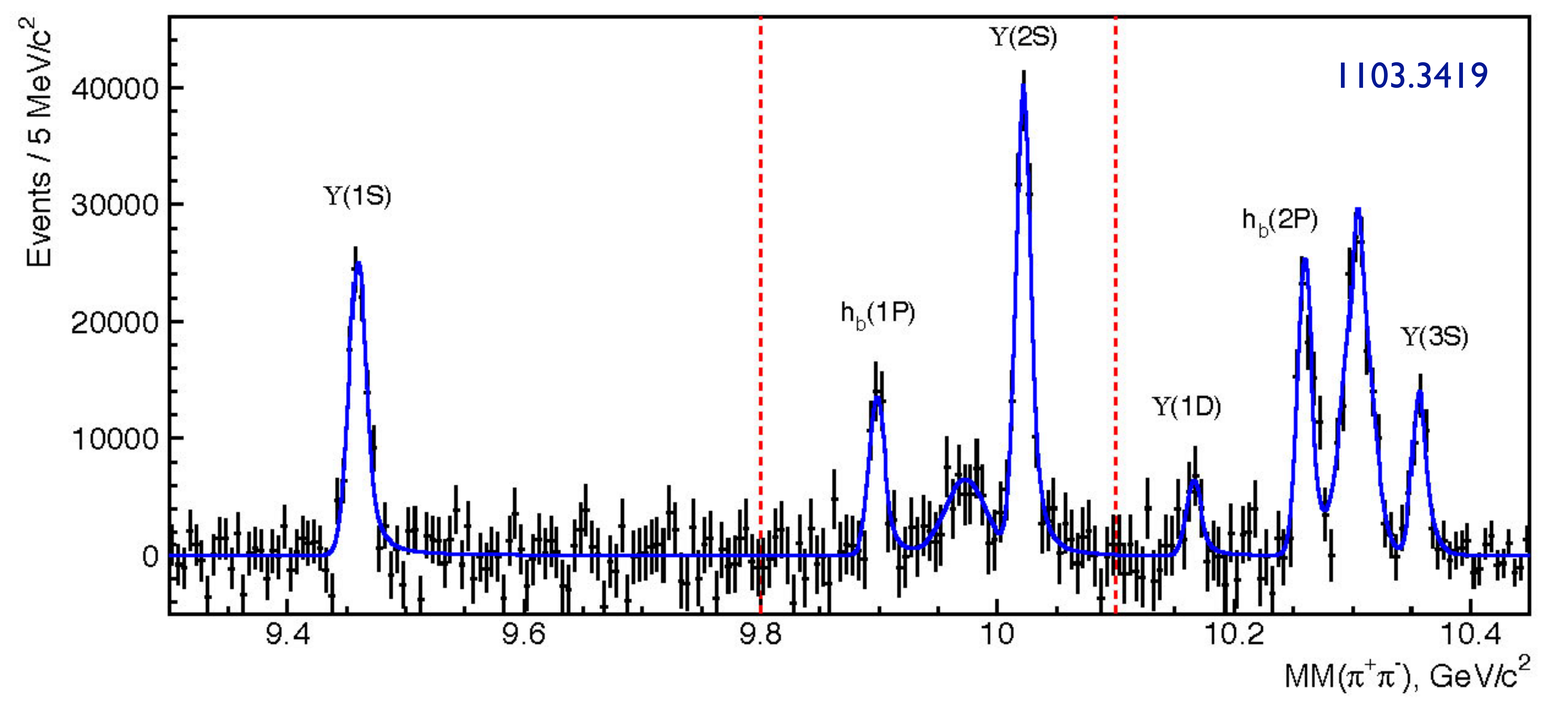
Bottomonium (PDG)



BaBar h_b(1P): $\Upsilon(3S) \rightarrow \pi^0 h_b(1P) \rightarrow \gamma\eta_b(1S)$



BELLE h_b(1P) and h_b(2P): $\pi^+\pi^- + \text{MM}$



New States Associated with Charmonium

- $X(3872) \to \pi^+\pi^- J/\psi$ at $D^0\bar{D}^{*0}$ threshold;
  very likely $J^{PC} = 1^{++}$;
  $(c\bar{c})$ + s-wave threshold + "molecule" + ...?
  Feshbach resonances? coupled-channels
- $Z(3930)$ in $\gamma\gamma \to D\bar{D}$: χ'_{c2}
- $X(3940)$ in $e^+e^- \to J/\psi D^* \bar{D}$
- $Y(3940)$ in $B \to K\omega J/\psi$
- $X(3915)$ in $\gamma\gamma \to \omega J/\psi \sim Z(3930)$?
- $Y(4140)$ in $J/\psi\phi$
- $Y(4260)$ in $e^+e^- \to \gamma_{\text{ISR}} J/\psi \pi^+\pi^-$
- $Z^+(4430) \to \pi^+\psi'$: not yet confirmed

2^{--}, 2^{-+} states (above $D\bar{D}$, but narrow) still missing


富士山